Planning of Manufacturing and Distribution in a Global Corporation

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Introduction

This paper is concerned with the planning of manufacturing and distribution in a global corporation. We make the assumption that we are systems analysts of a consulting firm whose clients are global corporations that need to decrease costs in manufacturing of their products. Hereon we present a mathematical description of this challenge as a *problem formulation*, an attempt to identify our customer's problematic state, namely the desire to maximize profitability without concrete methods to achieve this goal, and to provide them a mathematical model that serves as a suitable reference for future implementations.

Model

We model this system as a collection of internal and external suppliers of composite and atomic parts that comprise a final assembly. Cost considerations include production costs, transportation costs, and operational fees.

Two independent cases arise, suppliers managed by the corporation and suppliers that are not. For the first case, there are n different locations to choose for building factories, and there are n tasks that need to be done in specialized factories. We consider an n element set $\mathbb{A} = \{1, 2, \dots, i, \dots, k, \dots, n\}$. This represents each individual task. And we consider another n element set $= \{1, 2, \dots, j, \dots, l, \dots, n\}$. This represents each individual location.

$T = (t_{ik})$, where t_{ik} is the transportation size from task i and j. (Units: Pieces)

$D = (d_{jl})$, where d_{jl} is the transportation cost from location j to l. (Units: \$/Pieces)

$P = (p_{ij})$, where $p_{ij}$is the cost price for task i to be done at location j. (Units: \$/Pieces)

$C = (c_{ij})$, where c_{ij} is the factory construction fee for task i and location j. (Units: \$)

We define an indicator random variable $x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ such that if task i is assigned to location j, then $x_{ij} = 1$; if task i is not assigned to location j, then $x_{ij} = 0$.

$$\mathbb{O} = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} t_{ik} \cdot (d_{jl} + p_{ij}) \cdot x_{ij} x_{kl} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (1)$$

For the second case, We consider an m element set $\mathbb{C} = \{1, 2, \dots, a, \dots, m\}$. This represents the tasks need to be done by an already-existing manufacturer. We consider an o element set $\mathbb{D} = \{1, 2, \dots, b, \dots, o\}$. This represents the manufactures that are capable of performing any of these tasks.

$T' = (t'_{ai})$, where t'_{ai} is the transportation size from manufacturer task a to task i.

$D' = (d'_{bj})$, where d'_{bj} is the transportation cost from manufacture b to location j.

$P' = (p'_{ab})$, where p'_{ab} is the cost price for task a to be done at manufacturer b.

We define an indicator random variable $x'_{ab} = \begin{cases} 1 \\ 0 \end{cases}$ such that if manufacture a is assigned to task b, then $x'_{ab} = 1$; if manufacture a is not assigned to task i, then $x'_{ab} = 0$.

We still have the indicator random variable $x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ such that if task i is assigned to location j, then $x_{ij} = 1$; if task i is not assigned to location j, then $x_{ij} = 0$.

$$\mathbb{O} = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{a=1}^{m} \sum_{b=1}^{o} t'_{ai} \cdot (d'_{aj} + p'_{ab}) \cdot x'_{ab} x_{ij} \qquad (2)$$

Model (Continued)

Now that we have formal definitions for both case (1) and case (2), we can combine these definitions to produce a formula that describes the system as a whole.

$$\mathbb{O} = \min \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} t_{ik} \cdot (d_{jl} + p_{ij}) \cdot x_{ij} x_{kl} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{a=1}^{m} \sum_{b=1}^{o} t'_{ai} \cdot (d'_{aj} + p'_{ab}) \cdot x'_{ab} x_{ij} \right] \qquad (3)$$

Simplified Example

Let's consider three locations and three tasks. We assign each parameter a heuristic value as is shown in the following tables.

t_{ik}	A	B	C
A	—	10	7
B	0	—	13
C	20	0	—

d_{jl}	a	b	c
a	—	20	30
b	25	—	40
c	15	35	—

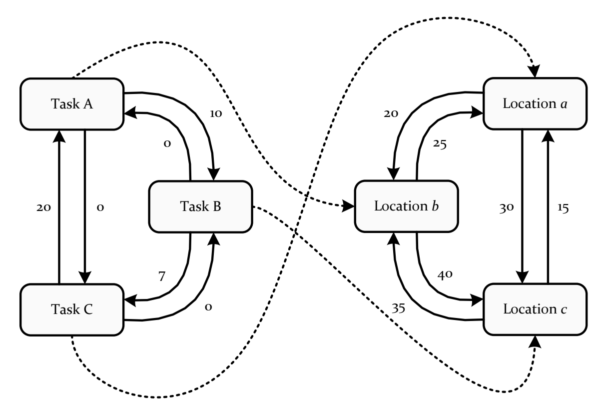
c_{ij}	A	B	C
a	200	700	1000
b	300	650	1200
c	400	600	900

p_{ij}	A	B	C
a	5	10	20
b	6	8	28
c	7	5	22

This yields six permutations of assignments, which we will call plans, and these plans are listed in the table below.

Plan 1	$A \to a$	$B \to b$	$C \to c$
Plan 2	$A \to a$	$B \to c$	$C \to b$
Plan 3	$A \to b$	$B \to a$	$C \to c$
Plan 4	$A \to b$	$B \to c$	$C \to a$
Plan 5	$A \to c$	$B \to a$	$C \to b$
Plan 6	$A \to c$	$B \to b$	$C \to a$

To find the optimal solution, we need to evaluate all plans in accordance with (3) and keep the minimum value obtained. An example for one such plan, Plan 4, is shown in the graph below.



$$\mathbb{O} = t_{AB} \cdot (d_{bc} + p_{bA}) + t_{AC} \cdot (d_{ba} + p_{bA}) + t_{BC} \cdot (d_{ca} + p_{cB}) + t_{BA} \cdot (d_{cb} + p_{cB}) + t_{CA} \cdot (d_{ab} + p_{cA}) + t_{CB} \cdot (d_{ac} + p_{cA}) + c_{bA} + c_{cB} + c_{cA} = 3637$$