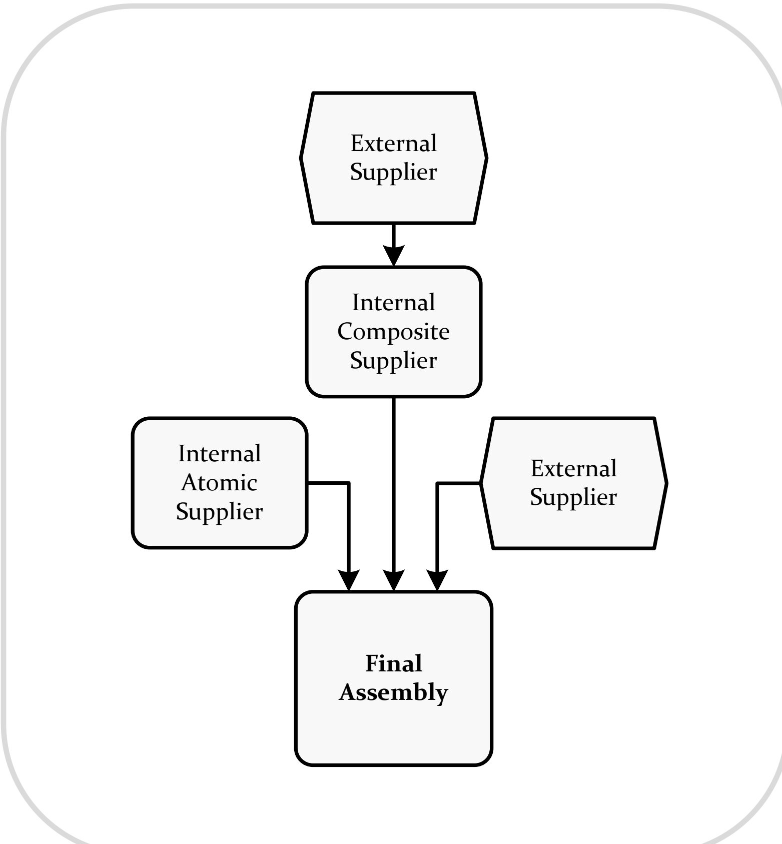
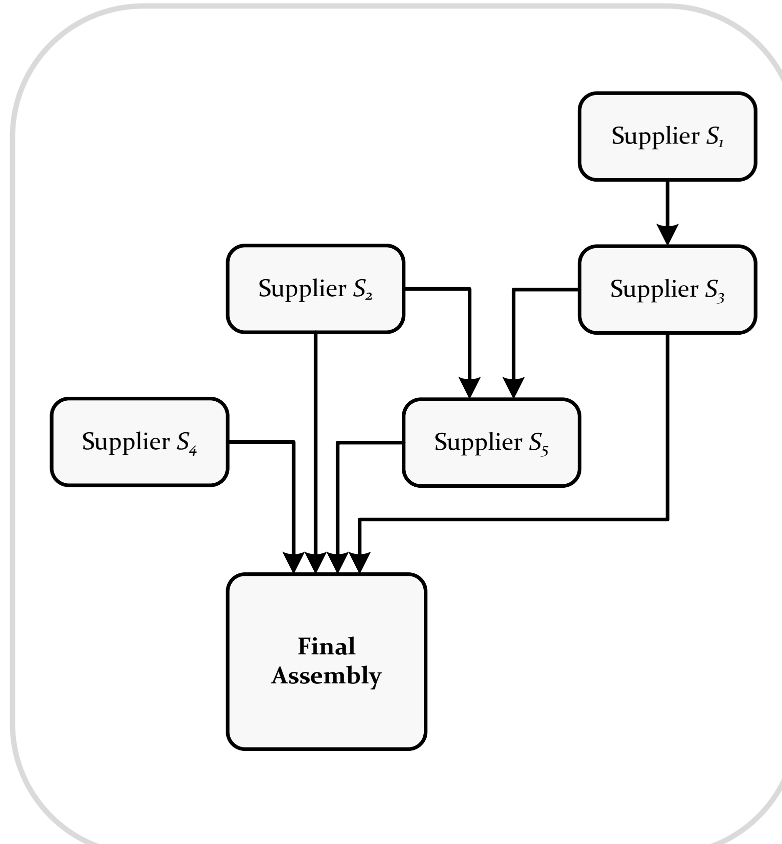
Problem Formulation

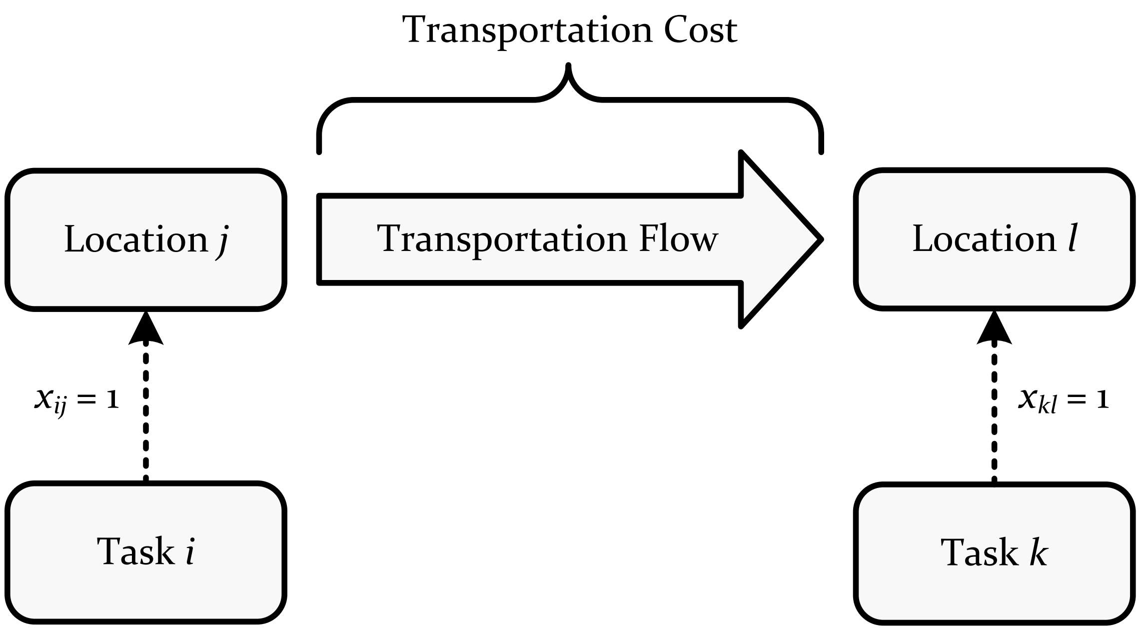
Organization of Parts and Manufacturing Facilities





Configuration Setup for the Model

Facilities Controlled by the Corporation



Configuration Setup for the Model

Facilities Controlled by the Corporation

Input

There are n different locations to choose for building factories, and there are n tasks that need to be done in specialized factories. We consider an n element set $\mathbb{A} = \{1, 2, \dots, i, \dots, k, \dots, n\}$. This represents each individual task. And we consider another n element set $\mathbb{B} = \{1, 2, \dots, j, \dots, l, \dots, n\}$. This represents each individual location.

$T = (t_{ik})$, where t_{ik} is the transportation size from task i and task j. The unit is Q pieces.

$D = (d_{jl})$, where d_{jl} is the transportation cost from location j to location l. The unit is K dollars per Q pieces.

$P = (p_{ij})$, where $p_{ij}$is the cost price for task i to be done at location j. The unit is K dollars per Q pieces.

$C = (c_{ij})$, where c_{ij} is the factory construction fee for task i and location j. The unit is K dollars.

We define an indicator random variable $x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ such that if task i is assigned to location j, then $x_{ij} = 1$; if task i is not assigned to location j, then $x_{ij} = 0$.

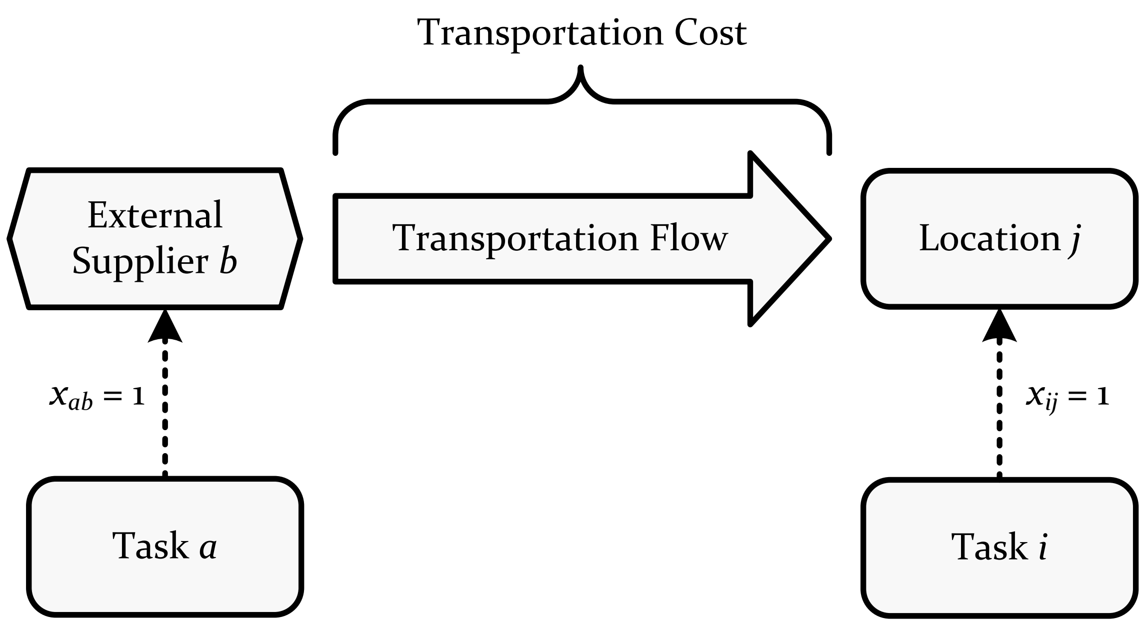
Output

This first case can be expressed as the minimum of the formula below.

$$\mathbb{O} = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} t_{ik} \cdot \left(d_{jl} + p_{ij}\right) \cdot x_{ij} x_{kl} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

Configuration Setup for the Model

Facilities Not Controlled by the Corporation



Configuration Setup for the Model

Facilities Not Controlled by the Corporation

Input

We consider an m element set $\mathbb{C} = \{1, 2, \dots, a, \dots, m\}$. This represents the tasks need to be done by an already-existing manufacturer. We consider an o element set $\mathbb{D} = \{1, 2, \dots, b, \dots, o\}$. This represents the manufactures that are capable of performing any of these tasks.

$T' = (t'_{ai})$, where t'_{ai} is the transportation size from manufacturer task a to task i. The unit is Q pieces.

$D' = (d'_{bj})$, where d'_{bj} is the transportation cost from manufacture b to location j. The unit is K dollar per Q pieces.

$P' = (p'_{ab})$, where p'_{ab} is the cost price for task a to be done at manufacturer b. The unit is K dollar per Q pieces.

We define an indicator random variable $x'_{ab} = \begin{cases} 1 \\ 0 \end{cases}$ such that if manufacture a is assigned to task b, then $x'_{ab} = 1$; if manufacture a is not assigned to task i, then $x'_{ab} = 0$.

We still have the indicator random variable $x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ such that if task i is assigned to location j, then $x_{ij} = 1$; if task i is not assigned to location j, then $x_{ij} = 0$.

Output

This second case can be expressed as the minimum of the formula below.

$$\mathbb{O} = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{a=1}^{m} \sum_{b=1}^{o} t'_{ai} \cdot \left(d'_{aj} + p'_{ab}\right) \cdot x'_{ab} x_{ij} \qquad (2)$$

Configuration Setup for the Model

Combined Cases

$$\mathbb{O} = \min\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} t_{ik}\cdot\left(d_{jl}+p_{ij}\right)\cdot x_{ij}x_{kl} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{a=1}^{m}\sum_{b=1}^{o} t'_{ai} \cdot\left(d'_{aj}+p'_{ab}\right)\cdot x'_{ab}x_{ij}\right]$$

Simple Example

Heuristic Values

Internal Relationship of Three Tasks

t_{ik}	A	B	C
A	—	10	7
B	0	—	13
C	20	0	—

Operational Fees at Different Locations

c_{ij}	A	B	C
a	200	700	1000
b	300	650	1200
c	400	600	900

Internal Relationship of Three Locations

d_{jl}	a	b	c
a	—	20	30
b	25	—	40
c	15	35	—

Cost Price for Locations to Finish Tasks

p_{ij}	A	B	C
a	5	10	20
b	6	8	28
c	7	5	22

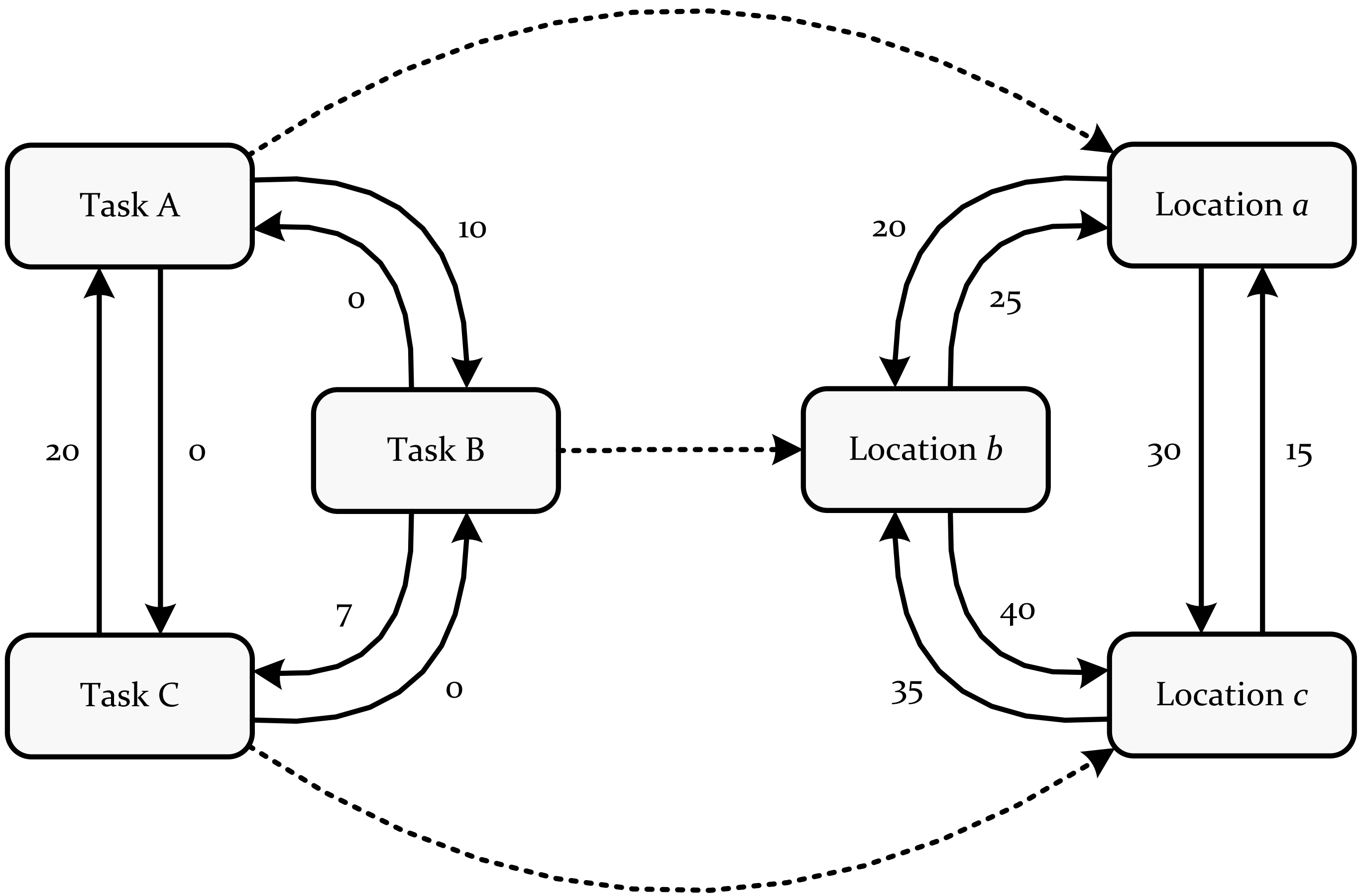
Simple Example

Plans of Action

Plan 1	$A \to a$	$B \to b$	$C \to c$
Plan 2	$A \to a$	$B \to c$	$C \to b$
Plan 3	$A \to b$	$B \to a$	$C \to c$
Plan 4	$A \to b$	$B \to c$	$C \to a$
Plan 5	$A \to c$	$B \to a$	$C \to b$
Plan 6	$A \to c$	$B \to b$	$C \to a$

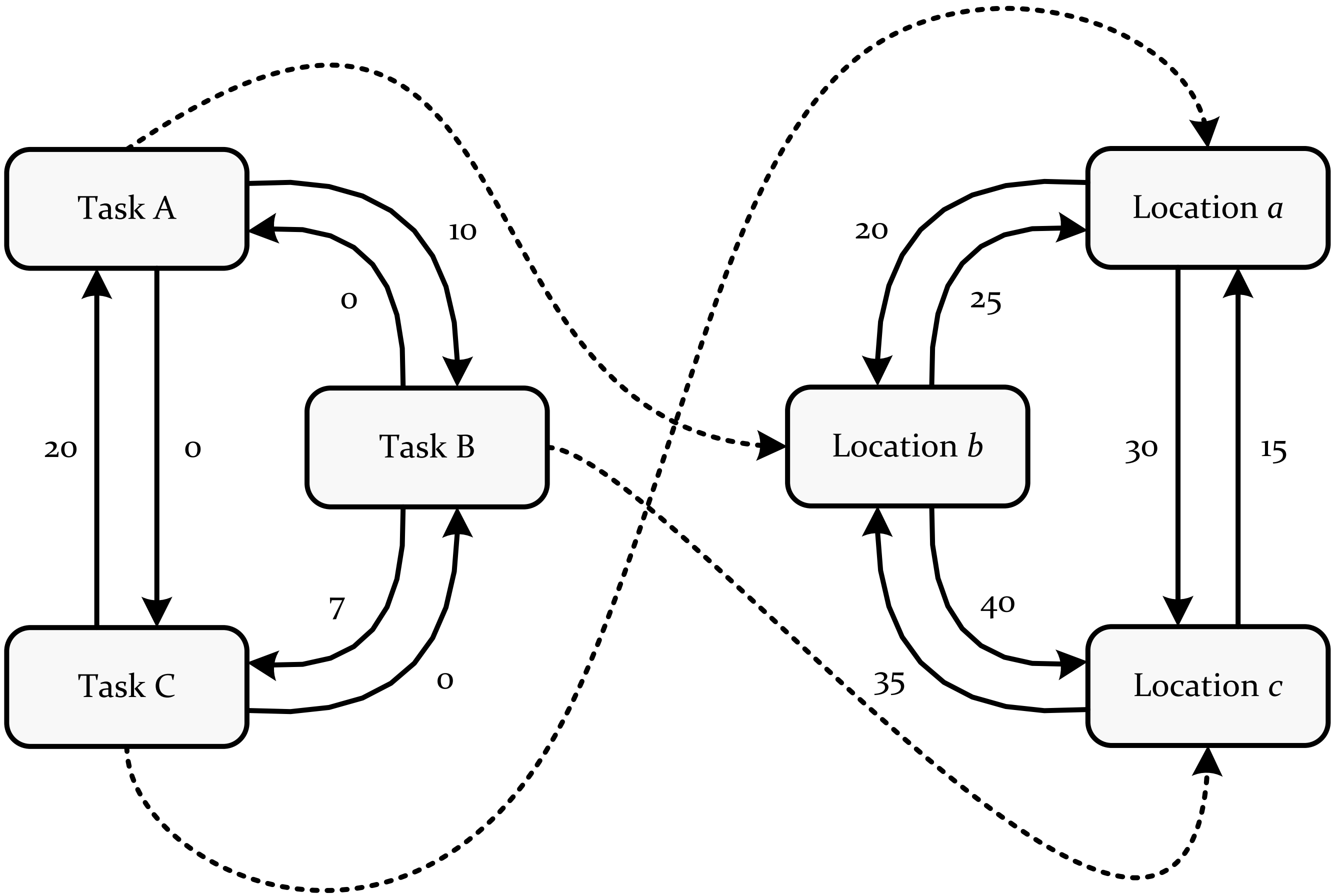
Simple Example

Visualization of the Model – Plan 1



Simple Example

Visualization of the Model – Plan 4



Simple Example

Minimization

Plan 1

$$\mathbb{O} = t_{AB} \cdot (d_{ab} + p_{aA}) + t_{AC} \cdot (d_{ac} + p_{aA}) + t_{BC} \cdot (d_{bc} + p_{bB}) + t_{BA} \cdot (d_{ba} + p_{bB}) + t_{CA} \cdot (d_{ca} + p_{cC}) + t_{CB} \cdot (d_{cb} + p_{cC}) + c_{aA} + c_{bB} + c_{cC}$$

$$= 10 \times (5 + 20) + 7 \times (5 + 30) + 13 \times (8 + 40) + 20 \times (22 + 15) + 200 + 650 + 900$$

$$= 3609$$

Plan 4

$$\mathbb{O} = t_{AB} \cdot (d_{bc} + p_{bA}) + t_{AC} \cdot (d_{ba} + p_{bA}) + t_{BC} \cdot (d_{ca} + p_{cB}) + t_{BA} \cdot (d_{cb} + p_{cB}) + t_{CA} \cdot (d_{ab} + p_{cA}) + t_{CB} \cdot (d_{ac} + p_{cA}) + c_{bA} + c_{cB} + c_{cA}$$

$$= 10 \times (6 + 40) + 7 \times (6 + 25) + 13 \times (5 + 15) + 20 \times (20 + 20) + 300 + 600 + 1000$$

$$= 3637$$

Concluding Remarks

- An ***optimal solution*** model is built to serve as a future implementation reference for our global corporation client in order to achieve the most cost-effective manufacturing and distribution business planning.
- The model is constructed under a series of ***preliminaries***. We considered most of the possible factors affecting optimization in models like our own, but we analyzed only the key conceptions and limitations in depth.
- The problem is formulated as a ***weighted bipartite graph*** representing facility layout and location planning for both the internal and external suppliers of the corporation. The formulas allow for optimization by minimizing sums based on the weights of the graph.
- Neither an algorithm nor an implementation was provided, and as such, the model is not meant to be used directly in real life. Instead it is meant to serve as a ***simplified model*** of a real-world problem.