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ICS 311
Homework 13
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Solution for lecture 18

Exercise 29.5-5 on p. 817

Solve the following linear program using SIMPLEX

Maximize $x_1 + 3x_2$

Subject to
$$-x_1 + x_2 \leq -1$$
$$-2x_1 - 2x_2 \leq -6$$
$$-x_1 + 4x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Answer: To use the simplex method, we first need to convert the constraint inequalities to slack variables.

Standard form objective function $x_1 + 3x_2$

Standard form constraints
$$-x_1 + x_2 \leq -1$$
$$-2x_1 - 2x_2 \leq -6$$
$$-x_1 + 4x_2 \leq 2$$

Slack form objective function $x_1 + 3x_2 = z$

Slack form constraints
$$-x_1 + x_2 + s_1 = -1$$
$$-2x_1 - 2x_2 + s_2 = -6$$
$$-x_1 + 4x_2 + s_3 = 2$$

We now have three equations and 6 unknowns, x_1, x_2, s_1, s_2, s_3, and z. We also have 4 equations as shown above, one objective function and three constraints. The system has an infinite number of solutions since there are more unknowns than equations. We can make the system be consistent by assigning two, which is 6 – 4, of the variables a value of zero and then solving for the remaining 4 variables. This is accomplished by dividing the 5 variables (not including z) into two groups. They are Basic variables and Non-basic variables.

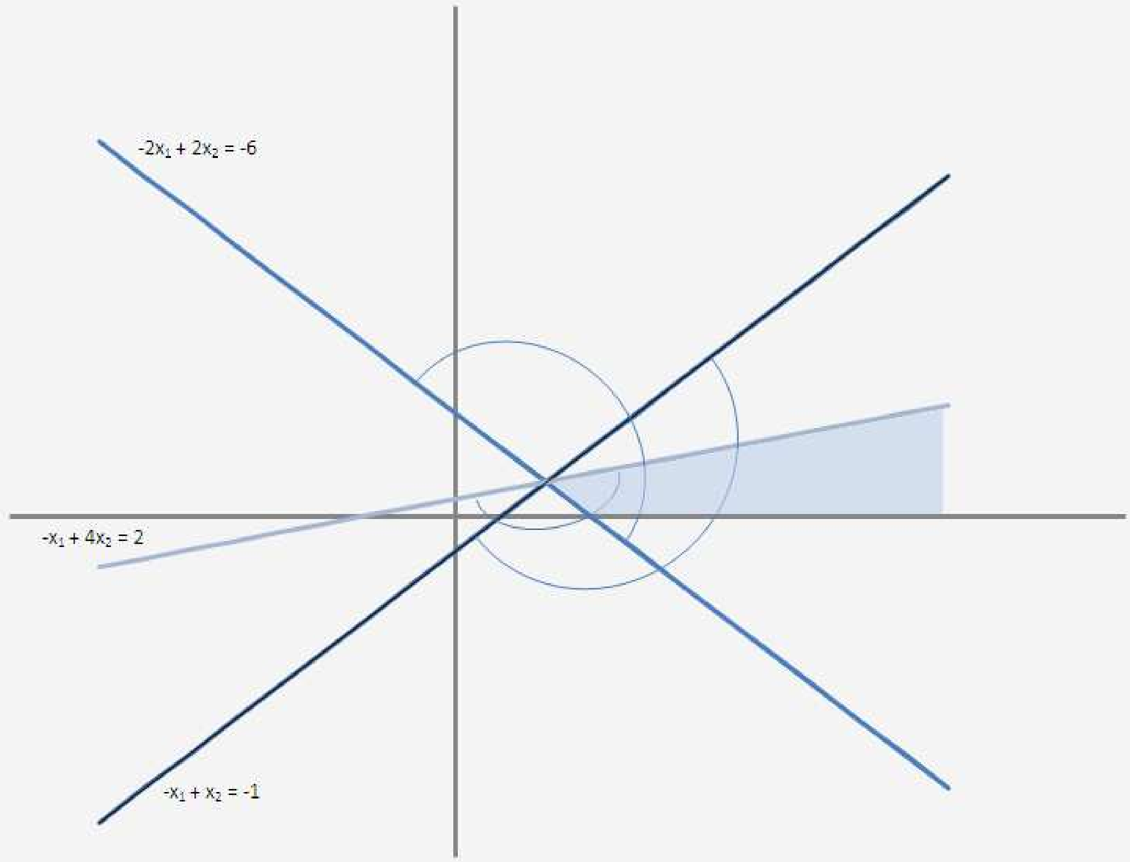
The selection is arbitrary. We assign any of the 2 variables to be non-basic variables and the other 3 to be basic variables. The non-basic variables are always assigned a value of zero. Then solve the equations for the basic solutions. We accomplish the overall optimization solution by repeating the pivot procedure and combinatory comparison.

Basic variables	Non-basic variables	x_1	x_2	s_1	s_2	s_3	z	Feasibility
s_1, s_2, s_3	x_1, x_2	0	0	-1	-6	2	N/A	No
x_2, s_2, s_3	x_1, s_1	0	-1	0	-4	6	N/A	No
x_2, s_1, s_3	x_1, s_2	0	3	-4	0	-10	N/A	No
x_2, s_1, s_2	x_1, s_3	0	0.5	-1.5	-5	0	N/A	No
x_1, s_2, s_3	x_2, s_1	1	0	0	-4	3	N/A	No
x_1, s_1, s_3	x_2, s_2	3	0	2	0	5	3	Yes
x_1, x_2, s_3	s_1, s_2	2	1	0	0	0	5	Yes
x_1, x_2, s_2	s_1, s_3	2	1	0	0	0	5	Yes
x_1, x_2, s_1	s_2, s_3	2	1	0	0	0	5	Yes

In this table, we observe that a basic solution that is not feasible includes at least one negative value and that a basic feasible solution does not include any negative values. The slack variables we introduced in earlier should be non-negative numbers based on the equations. The objective variables are both non-negative numbers based on the constraints.

At this point, according the simplex algorithm we found the maximum solution for vertices of the objective function. The maximum vertex value for objective function z is 5 according to the correctness of simplex algorithm. But from the table we can see that the feasible area is not a closed area because we need three lines to close a two dimensional shape while two of the three lines we had apparently meet at the same point. This is shown in the table that we got the same solutions for the last three combinations of equations.

We can determine the feasibility of a basic solution simply by examining the signs of all the variables in the solution. Observe that basic feasible solutions correspond to corner points (vertices) of the feasible region, which also include the optimum solution of the linear programming problems. Now, we put the solution in a graph form.



As shown in the graph, the feasible area is an open area with no bindery. Therefore we will not find a maximum solution for the objective function in this feasible area. Therefore we say the objective function does not have a solution with the given constraints.